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LETTER TO THE EDITOR

Exact solution of the Milburn equation for the two-photon Jaynes–Cummings model

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Abstract. An exact solution of the Milburn equation is given for the two-photon Jaynes–Cummings model of atom–field interaction with non-trivial dynamics. It is shown that the intrinsic decoherence in the atom–field interaction is responsible for the deterioration of the revivals of the atomic inversion.

Recently there has been considerable interest in understanding some fundamental problems in quantum mechanics [1–12]. It is well known that the feature of quantum mechanics that most distinguishes it from classical mechanics is the coherent superposition of distinct physical states. This is the basis for the concern about the quantum measurement [1, 2]. However, the superposition principle does not operate on a macroscopic scale, although nothing in the present formulation of quantum mechanics would indicate this.

Why does quantum coherence vanish on the macroscopic level? There are several approaches to solve the decoherence problem [4–11]. One of them is to modify the Schrödinger equation in such a way that coherence is automatically destroyed as the physical properties of the quantum system approach a macroscopic level. This intrinsic decoherence approach has recently been studied by several authors [7–11]. In particular, Milburn [11] has proposed a simple modification of standard quantum mechanics based on an assumption that on sufficiently short time steps the system evolves under unitary evolution in a stochastic sequence of identical unitary transformations. This assumption leads to a modified Schrödinger equation, called the Milburn equation, which contains a term responsible for the decay of quantum coherence.

Generally speaking, it is difficult to find an exact solution of the Milburn equation for a quantum system. In [11], Milburn considered only the evolution of a given free quantum system. Moya-Cessa *et al* [12] gave a form solution of the Milburn equation and studied the intrinsic decoherence in the atom–field interaction for the one-photon Jaynes–Cummings model (JCM). Recently, much attention has been paid to the quantum properties of multiphoton processes and the multiphoton laser both theoretically and experimentally [13–16]. It is noted that the quantum dynamics of the two-photon JCM is qualitatively different from that for the usual single-mode JCM. In this letter, we consider the two-photon JCM which is important in quantum optics. We will give an exact solution of the Milburn equation for the two-photon JCM and express explicitly the solution in the two-dimensional

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atomic basis. We will also study the influences of the intrinsic decoherence on the atomic inversion in the JCM.

Consider a quantum system described by the density operator $\hat{\rho}(t)$. Under the assumption that on sufficiently short time steps the system does not evolve continuously under evolution but rather in a stochastic sequence of identical unitary transformations, the density operator satisfies the equation [11]

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = \gamma\left\{\exp\left[-\frac{\mathrm{i}}{\hbar\gamma}\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{\mathrm{i}}{\hbar\gamma}\hat{H}\right] - \hat{\rho}(t)\right\} \tag{1}$$

where $\hat{H}$ is the Hamiltonian of the system, γ is a parameter which is equal to the mean frequency of the unitary step. In the limit $\gamma \to \infty$, equation (1) reduces to the ordinary von Neumann equation for the density operator.

Expanding equation (1) to first order in γ^{-1}, one can obtain the Milburn equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\frac{\mathrm{i}}{\hbar}[\hat{H}, \hat{\rho}] - \frac{1}{2\hbar^2\gamma}[\hat{H}, [\hat{H}, \hat{\rho}]]\,. \tag{2}$$

The Hamiltonian for the two-photon JCM in the rotating-wave approximation [15] is given by

$$\hat{H} = \hbar\omega a^{+}a + \tfrac{1}{2}\hbar\omega_0\sigma_3 + \lambda(\sigma_{+}a^2 + a^{+2}\sigma_{-}) \tag{3}$$

where a and a^{+} are field annihilation and creation operators and ω and ω_0 are the frequencies of the field mode and atomic transition, respectively. In this letter we take exact resonance between the field and atomic transition frequencies, i.e. $\omega_0 = 2\omega$, σ_3 and $\sigma_{\pm}$ are the Pauli spin matrices, λ is the real coupling constant. For the sake of simplicity, in the Hamiltonian (3) we ignore the term which describes the intensity-dependent Stark shift of two-levels arising due to the transition to an intermediate level.

We now start to find the exact solution for the density operator $\hat{\rho}(t)$ of the Milburn equation (2) applied to the Hamiltonian (3). we first introduce three auxiliary operators $\hat{R}$, $\hat{S}$ and $\hat{T}$ defined by

$$\exp(\hat{R}\tau)\hat{\rho}(t) = \sum_{k=0}^{\infty}\left(\frac{\tau}{\gamma}\right)^k\frac{1}{k!}\hat{H}^k\hat{\rho}(t)\hat{H}^k \tag{4}$$

$$\exp(\hat{S}\tau)\hat{\rho}(t) = \exp(-\mathrm{i}\hat{H}\tau)\hat{\rho}(t)\exp(\mathrm{i}\hat{H}\tau) \tag{5}$$

$$\exp(\hat{T}\tau)\hat{\rho}(t) = \exp\left(-\frac{\tau}{2\gamma}\hat{H}^2\right)\hat{\rho}(t)\exp\left(-\frac{\tau}{2\gamma}\hat{H}^2\right) \tag{6}$$

which lead to

$$\hat{R}\hat{\rho} = \frac{1}{\gamma}\hat{H}\hat{\rho}\hat{H} \qquad \hat{S}\hat{\rho} = -\mathrm{i}[\hat{H}, \hat{\rho}] \qquad \hat{T}\hat{\rho} = -\frac{1}{2\gamma}\{\hat{H}^2, \hat{\rho}\}\,. \tag{7}$$

From equations (2) and (7), we can write the formal solution of the Milburn equation as

$$\hat{\rho}(t) = \exp(\hat{R}t)\exp(\hat{S}t)\exp(\hat{T}t)\hat{\rho}(0) \tag{8}$$

where $\rho\hat{(0)}$ is the density operator of the initial atom-field system. We assume that initially the field is prepared in coherent states $|z\rangle$:

$$|z\rangle = \sum_{n=0}^{\infty}\exp\left(-\tfrac{1}{2}|z|^2\right)\frac{z^n}{\sqrt{n!}}|n\rangle \equiv \sum_{n=0}^{\infty}Q_n|n\rangle \tag{9}$$

and the atom was prepared in its excited states $|e\rangle$, so that

$$\hat{\rho}(0) = |z\rangle\langle z| \otimes |e\rangle\langle e|\,. \tag{10}$$

In a two-dimensional atomic basis the Hamiltonian (3) and its square can be expressed as a sum of diagonal and off-diagonal terms, respectively,

$$\hat{H} = \hat{H}_0 + \hat{H}_I \qquad [\hat{H}_0, \hat{H}_I] = 0 \qquad \hat{H}^2 = \hat{A} + \hat{B} \qquad [\hat{A}, \hat{B}] = 0 \tag{11}$$

with

$$\hat{H}_0 = \omega \begin{pmatrix} \hat{n} + 1 & 0 \\ 0 & \hat{n} - 1 \end{pmatrix} \qquad \hat{H}_I = \lambda \begin{pmatrix} 0 & a^2 \\ a^{+2} & 0 \end{pmatrix} \tag{12}$$

$$\hat{A} = \begin{pmatrix} \hat{\beta}_{n+1} & 0 \\ 0 & \hat{\beta}_{n-1} \end{pmatrix} \qquad \hat{B} = \begin{pmatrix} 0 & a^2(\hat{n} - 1) \\ (\hat{n} - 1)a^{+2} & 0 \end{pmatrix} \tag{13}$$

where $\hat{\beta}_n = \omega^2\hat{n}^2 + \lambda^2\hat{n}(\hat{n} + 1)$, and we have assumed exact resonance between the field and atomic transition frequencies.

Let

$$\hat{\rho}_2(t) = \exp(\hat{S}t)\exp(\hat{T}t)\hat{\rho}(0)\,. \tag{14}$$

From the definition of the auxiliary operators and the initial condition (10), we can find that

$$\begin{aligned} \hat{\rho}_2(t) &= \exp(\hat{S}t)\exp\left(-\frac{1}{2\gamma}\hat{H}^2 t\right)\hat{\rho}(0)\exp\left(-\frac{1}{2\gamma}\hat{H}^2 t\right) \\ &= \exp(-\mathrm{i}\hat{H}_I t)\exp\left(-\frac{t}{2\gamma}\hat{B}\right)\hat{\rho}_1(t)\exp\left(-\frac{t}{2\gamma}\hat{B}\right)\exp(\mathrm{i}\hat{H}_I t) \end{aligned} \tag{15}$$

where

$$\hat{\rho}_1(t) = |\Psi(t)\rangle\langle\Psi(t)| \otimes |e\rangle\langle e| \qquad |\Psi(t) = \exp\left(-\frac{t}{2\gamma}\hat{\beta}_{n+1}\right)|ze^{-\mathrm{i}\omega t}\rangle \tag{16}$$

we can write the operator $\exp(-(t/2\gamma)\hat{B})$ in the form

$$\exp\left(-\frac{t}{2\gamma}\hat{B}\right) = \begin{pmatrix} \hat{X}_{n+1}(t) & -a^2\frac{\hat{Y}_{n-1}(t)}{\sqrt{(\hat{n}-1)\hat{n}}} \\ -\frac{\hat{Y}_{n-1}(t)}{\sqrt{(\hat{n}-1)\hat{n}}}a^{+2} & \hat{X}_{n-1}(t) \end{pmatrix} \tag{17}$$

where

$$\hat{X}_n(t) = \cosh\left(\frac{\lambda\omega t}{\gamma}\sqrt{\hat{n}^3(\hat{n}+1)}\right) \qquad \hat{Y}_n(t) = \sinh\left(\frac{\lambda\omega t}{\gamma}\sqrt{\hat{n}^3(\hat{n}+1)}\right). \tag{18}$$

Similarly, one can express the operator $\exp(-\mathrm{i}\hat{H}_I t)$ in the two-dimensional atomic basis as

$$\exp(-\mathrm{i}\hat{H}_I t) = \begin{pmatrix} \hat{C}_{n+1}(t) & -\mathrm{i}\,a^2\frac{\hat{S}_{n-1}(t)}{\sqrt{(\hat{n}-1)\hat{n}}} \\ -\mathrm{i}\frac{\hat{S}_{n-1}(t)}{\sqrt{(\hat{n}-1)\hat{n}}}a^{+2} & \hat{C}_{n-1}(t) \end{pmatrix} \tag{19}$$

where the operators $\hat{C}_n(t)$ and $\hat{S}_n(t)$ are defined as

$$\hat{C}_n(t) = \cos(\lambda t\sqrt{\hat{n}(\hat{n}+1)}) \qquad \hat{S}_n(t) = \sin(\lambda t\sqrt{\hat{n}(\hat{n}+1)})\,. \tag{20}$$

Combining expressions (17) and (19), we find that

$$\exp(-\mathrm{i}\hat{H}_I t)\exp\left(-\frac{t}{2\gamma}\hat{B}\right) = \begin{pmatrix} \hat{W}_{n+1}(t) & \frac{\hat{V}_{n+1}(t)}{\sqrt{(\hat{n}+1)(\hat{n}+2)}}a^2 \\ a^{+2}\frac{\hat{V}_{n+1}(t)}{\sqrt{(\hat{n}+1)(\hat{n}+2)}} & \hat{W}_{n-1}(t) \end{pmatrix} \tag{21}$$

where the operators $\hat{V}_n$ and $\hat{W}_n$ are defined by

$$\hat{V}_n = -\hat{C}_n\hat{Y}_n - \mathrm{i}\hat{S}_n\hat{X}_n \qquad \hat{W}_n = \hat{C}_n\hat{X}_n + \mathrm{i}\hat{S}_n\hat{Y}_n . \tag{22}$$

Substituting (21) into (15), we can obtain an explicit expression for the operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}_2(t) = \begin{pmatrix} \hat{\Psi}_{11}(t) & \hat{\Psi}_{12}(t) \\ \hat{\Psi}_{21}(t) & \hat{\Psi}_{22}(t) \end{pmatrix} \tag{23}$$

where we have used the following operator:

$$\hat{\Psi}_{ij}(t) = |\Psi_i(t)\rangle\langle\Psi_j(t)| \qquad (i,j = 1,2) \tag{24}$$

with

$$|\Psi_1(t)\rangle = \hat{W}_{n+1}(t)|\Psi(t)\rangle \qquad |\Psi_2(t)\rangle = a^{+2}\frac{\hat{V}_{n+1}(t)}{\sqrt{(\hat{n}+1)(\hat{n}+2)}}|\Psi(t)\rangle . \tag{25}$$

Taking into account the definition of the auxiliary operator $\hat{R}$, we can write the action of the operator $\exp(\hat{R}t)$ on the operator $\hat{\rho}_2(t)$

$$\hat{\rho}(t) = \sum_{k=0}^{\infty}\left(\frac{t}{\gamma}\right)^k \frac{1}{k!}\hat{H}^k\hat{\rho}_2(t)\hat{H}^k . \tag{26}$$

This is the exact solution of the Milburn equation (2) with the two-photon Jaynes–Cummings Hamiltonian (3) and the initial condition (10). In practical use, one wishes to express explicitly the density operator in terms of its matrix elements, so that in what follows we evaluate the matrix elements of the density matrix in the two-dimensional atomic basis.

Since $\hat{H}_o$ commutes with $\hat{H}_I$, from (11) we have

$$\hat{H}^k = \sum_{m=0}^{k}\begin{pmatrix} k \\ m \end{pmatrix}\hat{H}_o^{k-m}\hat{H}_I^m . \tag{27}$$

It can be proved that when m is an even number, the off-diagonal term of the operator $\hat{H}_o^{k-m}\hat{H}_I^m$ vanishes, and we have

$$\sum_{m\in\text{even}}\begin{pmatrix} k \\ m \end{pmatrix}\hat{H}_o^{k-m}\hat{H}_I^m = \frac{1}{2}\begin{pmatrix} \hat{\varphi}_{n+1}^k + \hat{\phi}_{n+1}^k & 0 \\ 0 & \hat{\varphi}_{n+1}^k - \hat{\phi}_{n+1}^k \end{pmatrix} \tag{28}$$

while when m is an odd number, the diagonal term of $\hat{H}_o^{k-m}\hat{H}_I^m$ vanishes, and we have

$$\sum_{m\in\text{odd}}\begin{pmatrix} k \\ m \end{pmatrix}\hat{H}_o^{k-m}\hat{H}_I^m = \frac{1}{2}\begin{pmatrix} 0 & a^2\frac{\hat{\varphi}_{n+1}^k - \hat{\phi}_{n+1}^k}{\sqrt{\hat{n}(\hat{n}-1)}} \\ \frac{\hat{\varphi}_{n+1}^k - \hat{\phi}_{n+1}^k}{\sqrt{\hat{n}(\hat{n}-1)}}a^{+2} & 0 \end{pmatrix} \tag{29}$$

where the operators $\hat{\varphi}_{n+1}^k$ and $\hat{\phi}_{n-1}^k$ are defined by

$$\hat{\varphi}_n^k = [\omega\hat{n} + \lambda\sqrt{\hat{n}(\hat{n}+1)}]^k \qquad \hat{\phi}_n^k = [\omega\hat{n} - \lambda\sqrt{\hat{n}(\hat{n}+1)}]^k . \tag{30}$$

From (27), (28) and (29) it follows that

$$\hat{H}^k = \begin{pmatrix} \hat{f}_{n+1}^{(k)} & a^2\frac{\hat{g}_{n-1}^{(k)}}{\sqrt{\hat{n}(\hat{n}-1)}} \\ \frac{\hat{g}_{n-1}^{(k)}}{\sqrt{\hat{n}(\hat{n}-1)}}a^{+2} & \hat{f}_{n-1}^{(k)} \end{pmatrix} \tag{31}$$

with

$$\hat{f}_n^{(k)} = \tfrac{1}{2}(\hat{\varphi}_n^k + \hat{\phi}_n^k) \qquad \hat{g}_n^{(k)} = \tfrac{1}{2}(\hat{\varphi}_n^k - \hat{\phi}_n^k) . \tag{32}$$

Making use of (23) and (31) we can obtain

$$\hat{m}^{(k)}(t) \equiv \hat{H}^k \hat{\rho}_2(t) \hat{H}^k = \begin{pmatrix} \hat{m}_{11}^{(k)}(t) & \hat{m}_{12}^{(k)}(t) \\ \hat{m}_{21}^{(k)}(t) & \hat{m}_{22}^{(k)}(t) \end{pmatrix} \tag{33}$$

where the matrix elements are given by

$$\hat{m}_{11}^{(k)}(t) = \hat{f}_{n+1}^{(k)} \hat{\Psi}_{11}(t) \hat{f}_{n+1}^{(k)} + a^2 \hat{g}_{n+1}^{(k)'} \hat{\Psi}_{21}(t) \hat{f}_{n+1}^{(k)} + \hat{f}_{n+1}^{(k)} \hat{\Psi}_{12}(t) \hat{g}_{n-1}^{(k)} + a^2 \hat{g}_{n-1}^{(k)'} \hat{\Psi}_{22}(t) \hat{g}_{n-1}^{(k)'} a^{+2} \tag{34}$$

$$\hat{m}_{22}^{(k)}(t) = \hat{g}_{n-1}^{(k)'} a^{+2} \hat{\Psi}_{11}(t) a^2 \hat{g}_{n-1}^{(k)'} + \hat{f}_{n-1}^{(k)} \hat{\Psi}_{21}(t) a^2 \hat{g}_{n-1}^{(k)'} + \hat{g}_{n-1}^{(k)'} a^{+2} \hat{\Psi}_{12}(t) \hat{f}_{n-1}^{(k)} + \hat{f}_{n-1}^{(k)} \hat{\Psi}_{22}(t) \hat{f}_{n-1}^{(k)} \tag{35}$$

$$\hat{m}_{12}^{(k)}(t) = \hat{f}_{n+1}^{(k)} \hat{\Psi}_{11}(t) a^2 \hat{g}_{n-1}^{(k)'} + a^2 \hat{g}_{n-1}^{(k)'} \hat{\Psi}_{21}(t) a^2 \hat{g}_{n-1}^{(k)'} + \hat{f}_{n+1}^{(k)} \hat{\Psi}_{12}(t) \hat{f}_{n-1}^{(k)} + a^2 \hat{g}_{n-1}^{(k)'} \hat{\Psi}_{22}(t) \hat{f}_{n-1}^{(k)} \tag{36}$$

$$\hat{m}_{21}^{(k)}(t) = \hat{g}_{n-1}^{(k)'} a^{+2} \hat{\Psi}_{11}(t) \hat{f}_{n+1}^{(k)} + \hat{f}_{n-1}^{(k)} \hat{\Psi}_{21}(t) \hat{f}_{n+1}^{(k)} + \hat{g}_{n-1}^{(k)'} a^{+2} \hat{\Psi}_{12}(t) \hat{g}_{n-1}^{(k)'} a^{+2} + \hat{f}_{n-1}^{(k)} \hat{\Psi}_{22}(t) \hat{g}_{n-1}^{(k)'} a^{+2} \tag{37}$$

with

$$\hat{g}_n^{(k)'} = \hat{g}_n^{(k)} ((\hat{n}+1)\hat{n})^{-1/2} . \tag{38}$$

From (26) and (33) we finally arrive at the explicit expression of the solution of the Milburn equation (2) for the two-photon JCM:

$$\hat{\rho}(t) = \begin{pmatrix} \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{t}{\gamma}\right)^k \hat{m}_{11}^{(k)}(t) & \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{t}{\gamma}\right)^k \hat{m}_{12}^{(k)}(t) \\ \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{t}{\gamma}\right)^k \hat{m}_{21}^{(k)}(t) & \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{t}{\gamma}\right)^k \hat{m}_{22}^{(k)}(t) \end{pmatrix} . \tag{39}$$

It is well known that in the JCM the quantum coherences which are built up during the interaction with the atom significantly affect the dynamics of the atom [17]. It is because of these coherences that one can observe collapses and revivals of the atomic inversion [18]. The intrinsic decoherence will suppress the revivals. To see this, we evaluate explicit expression for the time evolution of the atomic inversion defined by

$$W(t) = \mathrm{Tr}[\hat{\rho}(t)\sigma_3] . \tag{40}$$

With the help of the expression $\hat{H}\sigma_3 = \sigma_3(\hat{H}_o - \hat{H}_I)$, we rewrite (40) as

$$W(t) = \mathrm{Tr}\left\{ \exp\left[\frac{t}{\gamma}(\hat{H}_o^2 - \hat{H}_I^2)\right] \hat{\rho}_2(t)\sigma_3 \right\} . \tag{41}$$

It is easy to get that

$$\exp\left[\frac{t}{\gamma}(\hat{H}_o^2 - \hat{H}_I^2)\right] = \begin{pmatrix} \exp\left(\frac{t}{\gamma}\hat{\theta}_{n+1}\right) & 0 \\ 0 & \exp\left(\frac{t}{\gamma}\hat{\theta}_{n-1}\right) \end{pmatrix} \tag{42}$$

with

$$\hat{\theta}_n = \omega^2 \hat{n}^2 - \lambda^2 \hat{n}(\hat{n}+1) . \tag{43}$$

Substitution of (22) and (41) into (41) yields that

$$W(t) = \sum_{n=0}^{\infty} \left[\langle n| \exp\left(\frac{t}{\gamma}\hat{\theta}_{n+1}\right) \hat{\Psi}_{11}(t)|n\rangle - \langle n| \exp\left(\frac{t}{\gamma}\hat{\theta}_{n-1}\right) \hat{\Psi}_{22}(t)|n\rangle \right]. \tag{44}$$

It is straightforward to evalutate the two terms on the LHS in (44), their results are, respectively,

$$\begin{aligned}
&\langle n| \exp\left(\frac{t}{\gamma}\hat{\theta}_{n+1}\right) \hat{\Psi}_{11}(t)|n\rangle \\
&\quad = |Q_n|^2 \left\{ \cos^2\left[\lambda t\sqrt{(n+1)(n+2)}\right] \cosh^2\left[\frac{\lambda\omega t}{\gamma}\sqrt{(n+1)^3(n+2)}\right] \right. \\
&\quad \left. - \sin^2\left[\lambda t\sqrt{(n+1)(n+2)}\right] \sinh^2\left[\frac{\lambda\omega t}{\gamma}\sqrt{(n+1)^3(n+2)}\right] \right\} \\
&\quad \times \exp\left[-\frac{2\lambda^2\omega^2 t}{\gamma}(n+1)(n+2)\right]
\end{aligned} \tag{45}$$

$$\begin{aligned}
&\langle n| \exp\left(\frac{t}{\gamma}\hat{\theta}_{n-1}\right) \hat{\Psi}_{22}(t)|n\rangle \\
&\quad = |Q_n|^2 \left\{ \sin^2\left[\lambda t\sqrt{(n+1)(n+2)}\right] \cosh^2\left[\frac{\lambda\omega t}{\gamma}\sqrt{(n+1)^3(n+2)}\right] \right. \\
&\quad \left. + \cos^2\left[\lambda t\sqrt{(n+1)(n+2)}\right] \sinh^2\left[\frac{\lambda\omega t}{\gamma}\sqrt{(n+1)^3(n+2)}\right] \right\} \\
&\quad \times \exp\left[-\frac{2\lambda^2\omega^2 t}{\gamma} n(n-1)\right].
\end{aligned} \tag{46}$$

Substituting (45) and (46) into (44), we finally arrive at the result

$$W(t) = \sum_{n=0}^{\infty} |Q_n|^2 \exp\left[-\frac{2\lambda^2 t}{\gamma}(n+1)(n+2)\right] \cos\left[2\lambda t\sqrt{(n+1)(n+2)}\right] \tag{47}$$

where the probablity amplitudes Q_n are given by (9). We see that (47) in the limit $\gamma \to \infty$ reduce to the well known expression [15] for the atomic inversion in the two-photon JCM governed by the von Neumann equation.

The solution (47) indicates that in the evolution the additional term in the Milburn equation, which destroys quantum coherences, leads to the appearance of decay factors $\exp[-(2\lambda^2 t/\gamma)(n+1)(n+2)]$ in (47), which are responsible for the destruction of revivals of the atomic inversion. With the decrease of the parameter γ, i.e. with a more rapid decoherence, we can observe rapid deterioration of revivals of atomic inversion.

It is interesting to study further the influence of the intrinsic decoherence on other non-classical effects of light field in the JCM.

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